DERIVATION OF IDEAL DIODE EQUATION

PROF. DEVRAJ SINGH
RBIPSSR, VBSPU, JNP

To derive ideal diode (P-N) I-V characteristics following assumptions are made:

(i) The depletion region has abrupt boundaries and outside the boundaries, the semiconductor is assumed to be neutral.

(ii) The carrier densities at the boundaries are related by the electrostatic potential difference across the junction.

(iii) The low-injection condition, that is, the injected minority carrier densities, are small compared with the majority carrier densities; in other words, the majority carrier densities are changed negligibly at the boundaries of neutral regions by the applied bias; and

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(iv) Neither generation or recombination current exists in the depletion region, and the electron and hole currents are constant throughout the depletion region.

At thermal equilibrium, the majority carrier density in the neutral regions is essentially equal to the doping concentration. We use the subscripts n and p to denote the semiconductor type and the subscript 0 to specify the condition of thermal equilibrium. Hence n_{n0} and n_{p0} are the equilibrium electron densities in the N- and P- sides, respectively. The expression for the built-in potential can be rewritten as

$$V_{bi} = \frac{k_B T}{q} \log_e \frac{p_{p0} n_{n0}}{n_i^2} = \frac{k_B T}{q} \log_e \frac{n_{n0}}{n_{p0}} \rightarrow (1)$$

where the mass action law $p_{p0} n_{p0} = n_i^2$ has been used.

Rearranging Eq. (1) gives

$$n_{n0} = n_{p0} e^{\frac{qV_{bi}}{k_B T}} \longrightarrow (2)$$

Similarly, we have

$$p_{p0} = p_{n0} e^{\frac{qV_{bi}}{k_B T}} \longrightarrow (3)$$

We note from Eqs. (2) and (3) that the electron density and the hole density at the boundaries of the depletion region are related through the electrostatic potential difference V_{bi} at thermal equilibrium. From our second assumption we expect that the same relation holds when the electrostatic potential difference is changed by an applied voltage.

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When a forward bias is applied, the electrostatic potential difference is reduced to $V_{bi} - V_F$; but when a reverse bias is applied, the electrostatic potential difference is increased to $V_{bi} + V_R$. Thus, Eq. (2) is modified to n_n and n_p are the non-equilibrium electron densities at the boundaries of the depletion region in the N- and P-sides, respectively with V positive for forward bias and negative for reverse bias. For the low-injection condition, the injected minority carrier density is much smaller than the majority carrier density; therefore $n_n \simeq n_{n0}$. Substituting the condition of Eq. (2) into Eq. (3) yields the electron density at the boundary of the depletion region on the P-side ($x = -x_p$):

$$n_p = n_{p0} e^{\frac{qV}{k_B T}} \longrightarrow (4)$$

or $$n_p - n_{p0} = n_{p0}\left(e^{\frac{qV}{k_B T}} - 1\right) \longrightarrow (5)$$

Similarly

$$p_n = p_{n0} e^{\frac{qV}{k_B T}} \longrightarrow (6)$$

or $$p_n - p_{n0} = p_{n0}\left[e^{\frac{qV}{k_B T}} - 1\right] \longrightarrow (7)$$

at $x = x_n$ for the N-type boundary. Figures 1 (a) and (b) show band diagrams and carrier concentrations in a P-N junction under forward-bias and reverse-bias conditions respectively. Note that the minority

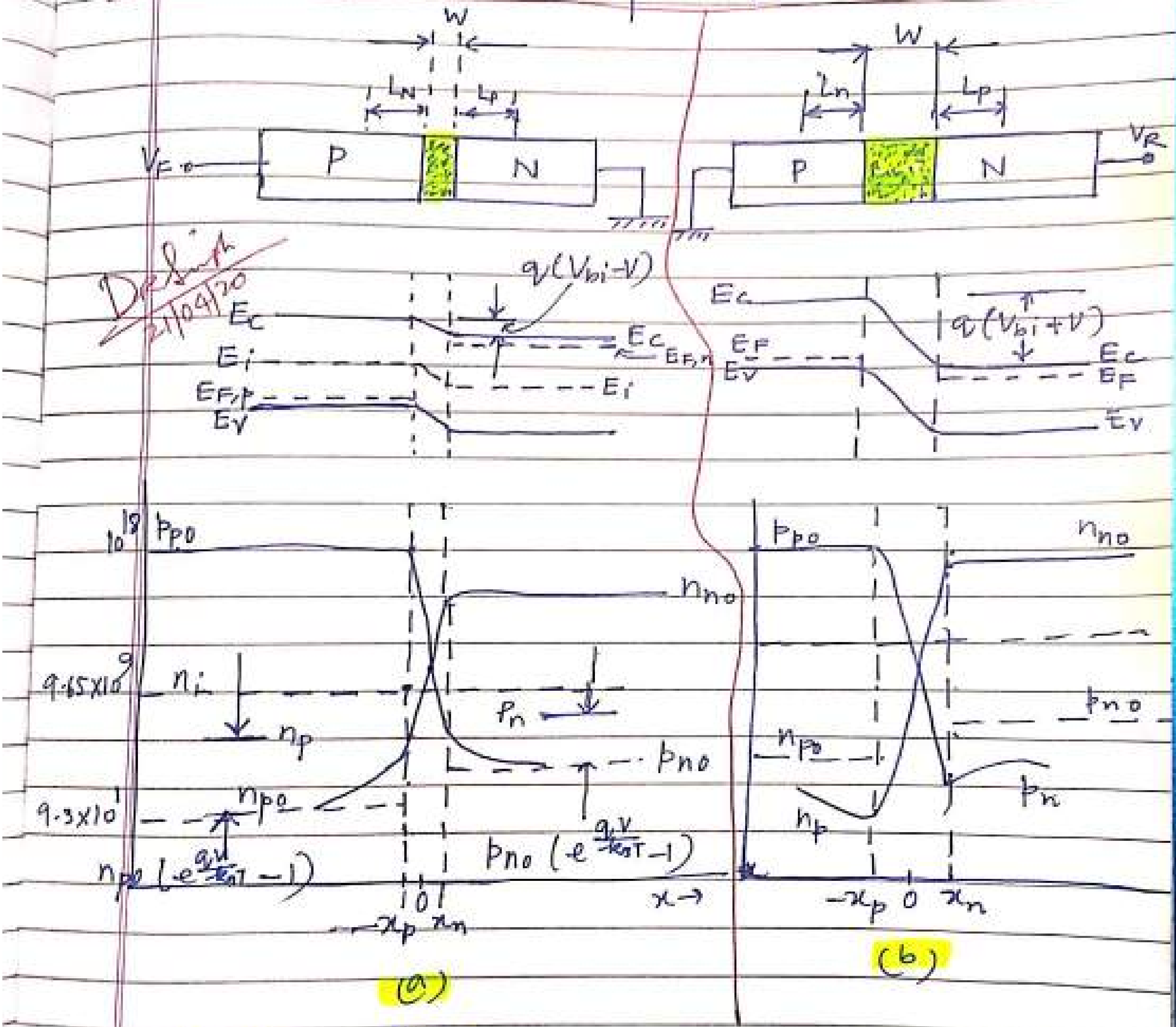

Fig. 1 Depletion region, energy band diagram and carrier distribution (a) forward bias, (b) reverse bias

carrier densities at the boundaries ($-x_p$ and x_n) increase ~~subsequently~~ substantially above their equilibrium values under forward bias, whereas they decrease below their equilibrium values under reverse bias. Equations (4) and (6) define the minority carrier densities at the boundaries of depletion region. These equations are the most important boundary conditions for the ideal current-voltage characteristics.

Under our idealized assumptions, no current is generated within the depletion region; all currents come from the neutral regions. In the neutral N-region, there is no electric field, thus the steady-state continuity equation reduces to

$$\frac{d^2 p_n}{dx^2} - \frac{p_n - p_{n0}}{D_p \tau_p} = 0 \longrightarrow (8)$$

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The solution of Eq. (8) with the boundary conditions of Eq. (8) and $p_n(x=\infty) = p_{n0}$ gives

$$p_n - p_{n0} = p_{n0}\left(e^{\frac{qV}{k_BT}} - 1\right) e^{-(x-x_n)/L_p} \longrightarrow (9)$$

where L_p, which is equal to $\sqrt{D_p \tau_p}$, is the diffusion length of holes (minority carriers) in the N-region.

At $x = x_n$

$$J_p(x_n) = -qD_p \left.\frac{dp_n}{dx}\right|_{x_n} = \frac{qD_p p_{n0}}{L_p}\left(e^{\frac{qV}{k_BT}} - 1\right) \rightarrow (10)$$

Similarly, we obtain for the neutral P-region

$$n_p - n_{p0} = n_{p0}\left(e^{\frac{qV}{k_BT}} - 1\right) e^{(x+x_p)/L_n} \longrightarrow (11)$$

and

$$J_n(-x_p) = qD_n \left.\frac{dn_p}{dx}\right|_{-x_p} = \frac{qD_n p_{p0}}{L_n}\left(e^{\frac{qV}{k_BT}} - 1\right) \longrightarrow (12)$$

where L_n, which is equal to $\sqrt{D_n \tau_n}$ is the diffusion length of electrons. The minority carrier densities [Eqs. (9) and (11)] are shown in the middle of Fig. 2.

Figure 2 illustrate that the injected minority carriers recombine with the majority carriers as the minority carriers move away from the boundaries. The electron and hole currents are shown at the bottom of Fig. 2. The hole and electron currents at the boundaries are given by Eqs. (10) and (12) respectively.

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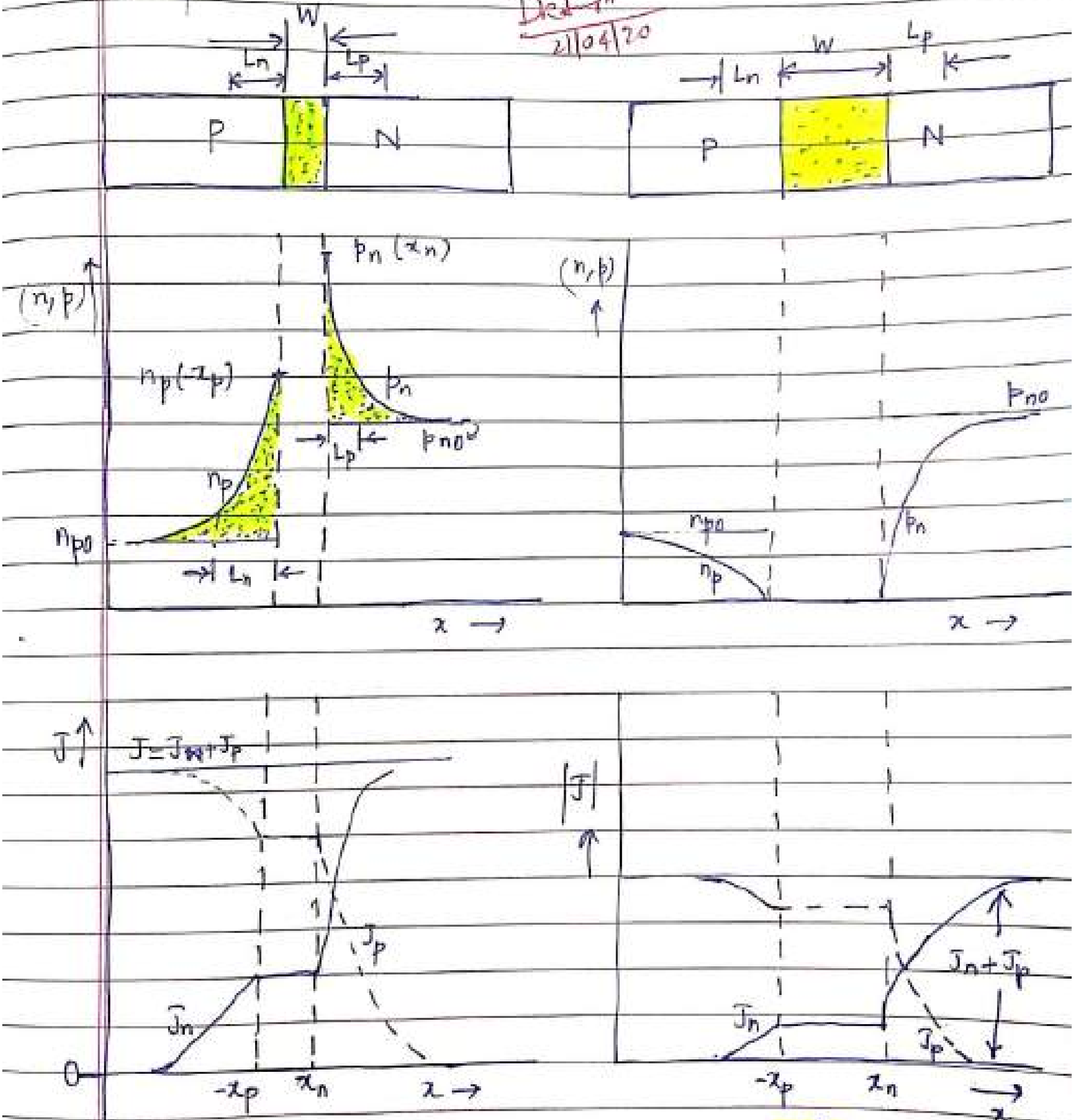

(a) (b)

Fig. 2. Injected minority carrier distribution and electron and hole currents (a) forward bias, (b) reverse bias

The hole diffusion current will decay exponentially in the N-region with diffusion length L_p, and the electron diffusion current will decay exponentially in the P-region with diffusion length L_n.

The total current is constant throughout the device and is the sum of Eqs (10) and (12)

$$J = J_p(x_n) + J_n(-x_p) = J_0\left[e^{\frac{qV}{k_BT}} - 1\right] \longrightarrow (13)$$

where
$$J_0 = \frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} \longrightarrow (14)$$

where J_0 is the saturation current density.

Equation (13) is the IDEAL DIODE EQUATION. The ideal current-voltage characteristic is shown in Fig. 2. In terms of current we have

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$$\text{As} \quad I_S = A \times J_0 \longrightarrow (15)$$

$$\therefore \quad I = I_0\left[e^{\frac{qV}{k_BT}} - 1\right] \longrightarrow (16)$$

$$\text{and} \quad I_0 = \frac{qAD_p p_{n0}}{L_p} + \frac{qAD_n n_{p0}}{L_n} \longrightarrow (17)$$

$$\boxed{\begin{aligned} I &= qA\left(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{p0}\right)\left(e^{qV/k_BT} - 1\right) \\ &= I_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] \end{aligned}} \longrightarrow (18)$$

where V is applied voltage and it can be positive and negative depending upon type of biasing.

→ In forward biasing, $V \Rightarrow$ positive and if $V > \frac{k_BT}{q}$, than $\exp\left(\frac{qV}{k_BT}\right) >> 1$. Hence $I = I_0 \exp\left(\frac{qV}{k_BT}\right)$, which means that I increases exponentially with forward bias.

In reverse biasing, V ⇒ negative, then $\exp.\frac{qV}{k_0T} \Rightarrow 0$

Hence $I = -I_0$ (directed from N to P)

NOTE:

Current flows relatively freely in the forward direction of the P-N junction diode, but almost no current flows in the reverse direction.

The characteristic curve (I-V) of ideal P-N junction diode is shown in Fig. 3

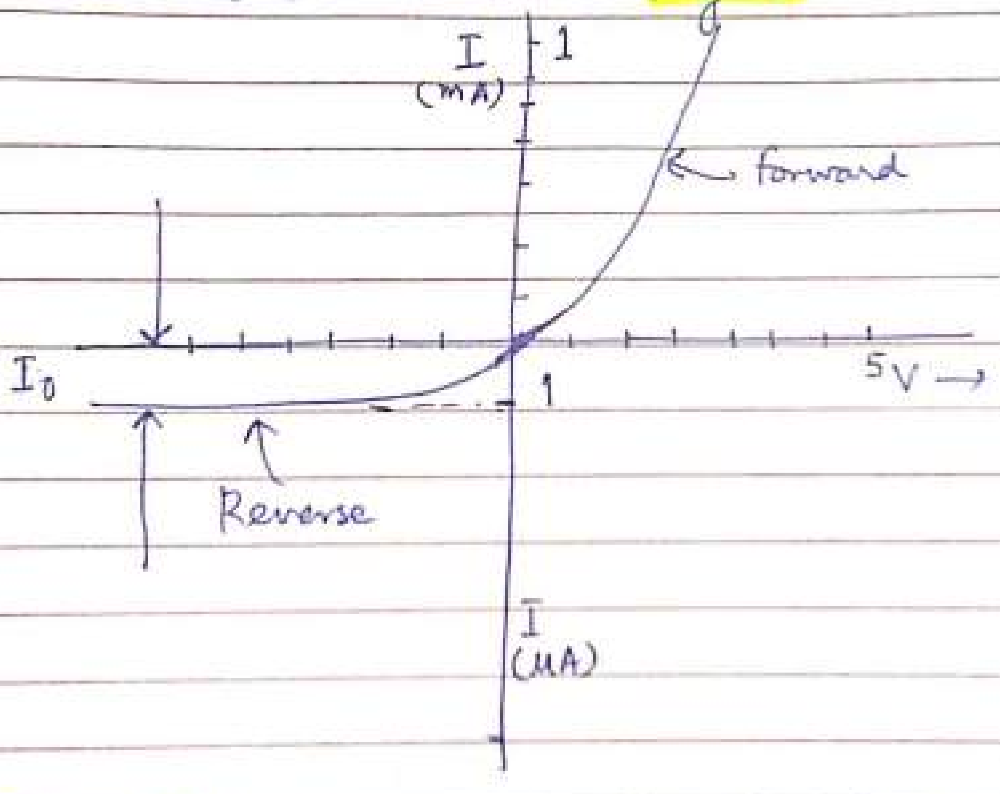

Fig. 3. Ideal current-voltage characteristics

NUMERICAL PROBLEMS

→ **PROBLEM #1**: Calculate the ideal reverse saturation current in a Si P-N junction diode with a cross-sectional area of $2\times10^{-4}\,cm^2$. The parameters of the diode are

$N_A = 5\times10^{16}\,cm^{-3}$, $N_D = 10^{16}\,cm^{-3}$, $n_i = 9.65\times10^{9}\,cm^{-3}$

$D_n = 21\,cm^2/s$, $D_p = 10\,cm^2/s$, $\tau_p = \tau_n = 5\times10^{-7}\,s$

SOLUTION: using

$$J_s = \frac{q D_p p_{n0}}{L_p} + \frac{q D_n n_{p0}}{L_n} = q n_i^2\left[\frac{1}{N_D}\sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_A}\sqrt{\frac{D_n}{\tau_n}}\right]$$

$$= 1.6\times10^{-19}\times(9.65\times10^{9})^2\left[\frac{1}{10^{16}}\sqrt{\frac{10}{5\times10^{-7}}} + \frac{1}{5\times10^{16}}\sqrt{\frac{21}{5\times10^{-7}}}\right]$$

$$= 8.58\times10^{-12}\,A/cm^2$$

From the cross-sectional area $A = 2\times10^{-4}\,cm^2$, we obtain

$$I_s = A\times J_s = 2\times10^{-4}\times8.58\times10^{-12}$$

$$= 1.72\times10^{-15}\,A.$$

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→ **PROBLEM #2**: The saturation current density of a P-N junction germanium diode is $250\,mA/m^2$ at $300K$. Find the voltage that would have to be applied across the junction to cause a forward current density of $10^5\,A/m^2$ to flow.

SOLUTION: We know that

$$I = I_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

and in terms of current density, we can write

$$J = J_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

or $$\exp\left(\frac{qV}{k_BT}\right) = \frac{J}{J_0} + 1 = \frac{10^5}{250\times10^{-3}} + 1$$

$$= 4\times10^5$$

$$\frac{qV}{k_BT} = \ln(4\times10^5) = 12.9$$

$$V = \frac{12.9\times1.38\times10^{-23}\times300}{1.6\times10^{-19}} = 0.33\,V$$